

# MATH AmSud

## 2022 call for proposal approved projects



### **CIPIF**

**Controllability and Inverse Problems in Free Boundary Problems for Incompressible Fluids**

### **GSA**

**Graph spectra and its applications**

### **GV-BCEF**

**The geometry of varieties: group actions, symmetries, moduli and beyond**

### **LESET**

**Lyapunov Exponents and Smooth Ergodic Theory**

### **MathNN4DE**

**Mathematical analysis of neural networks for solving partial differential equations and inverse problems.**

### **MEMSD**

**Modeling and estimating massive point referenced spatial data**

### **OCPROGEC**

**Optimality conditions and proximal methods for generalized convex optimization problems**

### **SCIPinPDEs**

**Stabilization, Control and Inverse Problems in PDEs**

### **STEMDYNEPID**

**Spatial and TEMPoral DYNamics of EPIDemiological model with age and spatial structures**

### **TOMCAT**

**Topological, combinatorial and probabilistic aspects in dynamical systems**

### **ATQHFRAG**

**Abstract theories of quadratic and Hermitian forms, and real algebraic geometry**

# Controllability and Inverse Problems in Free Boundary Problems for Incompressible Fluids

Our proposal focuses on mathematical models from the fluid mechanics, namely on incompressible fluids as those described by the water–waves equation, the Euler equations, the Navier–Stokes system, the Saint–Venant system and the KdV-equation. They include free-boundary problems, such as that of determining the free surface of the water-waves system of conservation laws, assuming it is known at time  $t = 0$ . They also include problems in which the Navier-Stokes system is coupled with different PDE's for studying heat thermal convection and the mixture of viscous fluids. In connection with the different models mentioned above, we pretend to study some control and inverse problems related. In this sense, some parameters as the pressure, velocity, density, viscosity or external forces must be considered when studying control or inverse problems. Moreover, depending on the domain shape, different boundary conditions may be considered, e.g., Dirichlet, Neumann, or free–slip, among others. The start will be a brief mathematical description of the model PDE's we consider. Then we formulate an optimal control problem in the shallow–water regime for the Saint–Venant–Boussinesq system and for the Green–Naghdi system. The goal is to recover the shape of the bottom surface in a channel by measurements on the free surface only. We also propose to implement a numerical strategy for the resolution of this optimal control problem; via a finite volume scheme that bears on the resolution of an adjoint problem. Next, we plan to study the null controllability property of the free surface for Euler's system, when the control function is localized on the rigid boundary of a given bounded domain. We will study a linearized version of that PDE, obtained by transformation into a non-local problem by means of a conformal map and by invoking the Hilbert transform, and we will implement a time reversal method to analyze the inverse problem of source detection. So far, we only considered PDE's that describe the motion of fluids due to applied boundary of force terms, without any retro action of the fluid on what generated these source terms. In many instances, one is interested in systems where the motion of the fluid is coupled with other physical phenomena. In this spirit, we also intend to consider a problem concerning the Navier-Stokes system coupled with heat transfer, or convection due to the mixing of fluids of different densities. Besides analyzing weak solutions to such system, we will focus on deriving error estimates and stability properties of associated Finite Elements discretizations.

## Project coordinators

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The present project proposes the establishment of a network of collaboration among Argentina, Chile, Brazil, and France, using the strength of 5 mathematics groups of 5 different institutions. The research topic of the proposal is Algebraic Graph Theory, an important and modern area of discrete mathematics. The proposal is structured in such a way that the training of highly qualified human resources and research activities are intertwined, this will ensure the generation of new knowledge in a relevant scientific area and leave permanent ties of collaboration between the different research groups beyond the completion of the project.

## Project coordinators

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Geometry has evolved from the axiomatic approach of the ancient mathematicians to the modern study of complex and real spaces.

Although this branch of Mathematics is commonly considered as a pure area, it has many applications in cryptography, coding theory, physics, differential equations, chemistry, among other areas.

This project is aimed at creating new mathematical networks and at - 1 - strengthening collaboration around geometry and related areas, by means of the organization of international schools for students, missions or research stays and workshops in Chile, Brazil, Ecuador and France.

This project has a special emphasis in involving and supporting young researchers and students.

The scientific goal is to understand of the geometry and symmetries of special algebraic varieties, in their many guises.

We propose four scientific activities: two schools for students in Brazil and Ecuador, a research workshop in France, and a final conference in Chile. All those activities will involve students and young researchers.

## Project coordinators

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A key feature of differentiable systems is the speed with which orbits separate as measured by its Lyapunov exponents. These numbers are defined pointwise and also at the level of ergodic invariant probability measures. They are related to dynamical, geometrical and analytical properties. However, - 1 - 2 these are asymptotic quantities, often discontinuous, and computing them is difficult and a central problem in smooth ergodic theory. In this project we will build on recent discoveries (e.g., entropy-continuity on surfaces, invariance principle for partially hyperbolic dynamics) to investigate properties of Lyapunov exponents and their consequences for higher dimensional systems (mostly of hyperbolic type but not only): entropycontinuity, geometry of invariant foliations (u-Gibbs or SRB measures).

## Project coordinators

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In the last years, numerical methods based on machine learning techniques have caught attention in researchers in many areas, due to their applicability. However, there is still missing a mathematical theory establishing important properties that we could expect from such a popular tool. In particular, recently some studies of deep learning methods for solving problems related to partial differential equations have been reported, but focusing mostly on experimental results.

We aim to study in a more theoretical way the properties of deep neural networks, mainly, the different error sources in the approximations of PDEs solutions by DNNs. Our proposal is based on the new area of mathematical foundations of deep learning; analysis of PDEs and inverse problems, and; numerical analysis.

## Project coordinators

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Random fields are the mathematical foundation for the statistical analyses of point-referenced spatial (multivariate) data which allows to describe the marginal behavior and to assess the dependence structure inside the data. Two fundamental steps needed when analyzing such data are modeling and estimation. These steps are challenging, in particular when analyzing massive spatial data.

The first proposal of our project focus in the estimation step and considers a highly scalable estimation method for (non-)Gaussian random fields. In particular, we focus on maximum weighted composite likelihood based on pairs (WCLP), a method of estimation that have been widely used for the estimation of random fields with (non-)Gaussian distribution. We propose a novel (a)symmetric weight function based on nearest neighbors for the WCLP method. The proposed weight function allows to estimate spatial massive (up to millions) datasets and to improve the statistical efficiency of the WCLP method. Regarding the second proposal we propose a novel class of non-Gaussian random fields named Tukey-hh random fields to model non-Gaussian spatial data. The proposed random fields have extremely flexible marginal distributions, possibly skewed and/or heavy-tailed, and, therefore, have a wide range of applications. The third proposal focus in the modeling step for multivariate Gaussian random fields. Specifically, we consider a general parsimonious specification for the cross-covariance matrix of a multivariate Gaussian random field that is based on Kronecker product and on the Cholesky decomposition of the marginal covariances matrices. The specification allows to reduce the number of parameters involved in the estimation step and to reduce the computational complexity of the model in particular when coupled with compactly supported correlation functions. Our developments will be implemented in an open-source package for the R statistical environment.

## Project coordinators

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# Optimality conditions and proximal methods for generalized convex optimization problems

This project is meant to contribute to the development of knowledge regarding optimization problems (in the largest sense) involving generalized convex functions. It is divided into five main objectives, each of them comprising of several intermediate goals.

In the framework of the first two objectives we will deal with optimization problems consisting in minimizing the sum of a (quasi)convex function with a strongly quasiconvex one. In the first case we will consider one of the functions to be differentiable, while in the second both of them will be taken nonsmooth. We plan to deliver optimality conditions for characterizing the solutions to these problems and splitting proximal point type algorithms for determining the latter.

We have similar goals for problems consisting in minimizing the difference of two such functions, too, where we will additionally consider subgradient type algorithms, too. The fourth objective of the project contains investigations on equilibrium problems, mixed variational inequalities and multiobjective optimization problems involving (strongly) quasiconvex functions. Last but not least we plan to deal with robust nonconvex quadratic optimization problems, too, for which robust optimality and alternative results are expected.

## Project coordinators

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The objectives of this project are divided into three parts depending on the type of partial differential equations we want to control or stabilize. The first part is devoted to the study of control properties of some parabolic systems, appearing, for example, in cardiovascular models but also for other parabolic equations with various constraints. In a second part, we propose controllability problems for systems of hyperbolic type such as elasticity, wave or plate equations. The last part concerns systems mixing hyperbolic and parabolic equations such as fluid-elastic interaction systems or equations with memory.

## Project coordinators

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# Spatial and TEMporal DYNamics of EPIDemiological model with age and spatial structures

Recent epidemiological outbreaks, including the recent COVID-19 but also Ebola, Zika and other pathogens, have highlighted the need for a better understanding of how pathogens spread and survive in a population. While each pathogen has a unique way of interacting with the hosts, some common patterns can be identified and studied in order to understand and better control how the epidemics behaves. Mathematical models precisely aim at building tools that can be adapted to understand individual pathogen's behavior given the need to do so. To that respect, the Kermack-McKendrick model with age of infection is particularly interesting as it allows to encode the pathogen shedding behavior of an host within the model. We propose to study the mathematical properties of KermackMcKendrick models with age of infection, that also have a spatial structure to represent the spatial component of the host population. Robust methods have been developed in the past to handle the spatial behavior of reaction-diffusion equations via the study of spreading speeds and traveling waves. We expect that the synergy between the two structures will give rise to interesting mathematical challenges but also to a better understanding of how pathogens spread in a population.

## Project coordinators

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# Topological, combinatorial and probabilistic aspects in dynamical systems structures

1. A long standing question posed by M.Shub is if for any degree  $d$  map of the two-sphere the number of periodic points increases exponentially as  $dn$ . Some simple examples show that this does not hold in general for continuous mappings, and it was conjectured that the answer is affirmative if the map is of class  $C^1$ . One of the objectives of this project is to attack the case of  $C^0$  branched coverings of the two-sphere, thus simplifying the structure of singularities but extending to non-smooth maps.
2. The study of rotation sets has been a useful tool in the classification of dynamics in low dimensions. This project considers some questions related to the relationship between dynamics and rotation sets. One of the objectives of this project is to understand the shape of rotation sets in higher genus compact surfaces. The case where the rotation set is a segment will be considered, as well as some new examples of rotation sets with nonempty interior.
3. Expansivity is central in dynamics, as a concept closely related to chaos. Most results consider compact ambient spaces, but the concept has multiple extensions to the noncompact case. One of our interests is to “compatibilize” definitions and establish a connection between the expansivity of a system on a noncompact space and its restrictions to compact invariant subspaces.
4. Piecewise contractions arise in dynamical systems as first return maps of Cherry flows and of strange or outer billiards. In applied mathematics, high dimensional piecewise contractions appear as models of biological networks. This project proposes to study the number, the combinatorial, and the ergodic properties of the attractors of interval piecewise contractions. In higher dimension, focus is made on the genericity of periodic attractors.
5. Extreme Value Theory aims to describe the statistics of rare events. It has been introduced in the context of dynamical systems two decades ago and is nowadays a very productive field. In this project, it is planned to study the statistics of extreme events for different classes of dynamical systems, namely: high dimensional invertible transformations, coupled maps, random open systems and piecewise contractions.
6. In many other areas of mathematics, it is often interesting to have a good estimate of the cardinality of a combinatorial set as a function of some parameter. In symbolic dynamics, an example is the number of words of any given size contained in the language of a sub-shift, or a given infinite word. This part of the project aims to use and improve a probabilistic method to study the asymptotics of several integer partitions functions and to classify the complexity functions of cyclic languages from symbolic dynamics.

## Project coordinators

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This project aims to pursue: (i) the study of the theory of quadratic forms over semireal (commutative, unitary) rings using the resources coming from the abstract algebraic configuration provided mainly by the (first-order) theories of special groups and real semigroups; (ii) the formulation and development of a new frontier: an abstract, and first-order, theory of the algebraic theory of hermitian forms; (iii) by the algebraic and model-theoretic tools developed, produce connections and applications to real algebraic geometry and o-minimality.

## Project coordinators

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