

MATH AMSUD SCIENTIFIC SEMINAR REPORT

NOV. 30TH 2021

Stochastic calculus, theory and applications



PRESENTATION OF THE MAIN RESULTS OF THE SEMINAR

ARGENTINA BOLIVIA BRASIL CHILE COLOMBIA ECUADOR FRANCIA PARAGUAY PERÚ URUGUAY VENEZUELA

PRESENTATION OF THE REPORT

This report has been produced based on the interventions of 9 mathematic specialists from South America and France during the MATH AmSud scientific seminar. Its objective is to present a sinthesis of the highlights of the seminar, to broaden the diffusion of results and tools presented among the scientific society and more generally among the civil society. It also suggests the next steps and potential for cooperation actions in South America that could be submitted in the framework of the MATH AmSud program.

The seminar was organized by the MATH AmSud secretary composed of the French Embassy in Chile and the Chilean Innovation and Development National Agency (ANID), the Uruguayan Innovation and research Agency (ANII) and the French Ambassy in Uruguay. 70 participants attended this seminar, from 11 countries,

The organizers of the seminar thank Pablo Muse, Marc Hindry, Fernando Paganini, Ernesto Mordecki, Jean-François Delmas, José Cerda Hernández, Roberto Viveros, Pablo Groisman, Alejandro Ramírez, Yuri Saporito and Viswanathan Arunachalam who made this report possible and who contributed to the success of the seminar.

MATH AmSud is one of the three « AmSud » programmes, which support regional mobility projects between France and South America countries. It is funded by 17 structures, listed on the last page of the report. More information at <u>www.sticmathamsud.org</u>

Watch the seminar https://www.youtube.com/watch?v=_I9wyN42v6o&t=34s



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FLUID AND DIFFUSION MODELS FOR PROVISIONING OF SERVERS IN THE CLOUD

FERNANDO PAGANINI UNIVERSIDAD ORT, URUGUAY

Content of the presentation

The theory of fluid and diffusion limits for Markov processes is applied to study performance of cloud computing systems which adapt their service capacity. We develop extensions of the classical results of Kurtz, required to accommodate non-homogeneous intensity scalings and non-differentiable drift fields. Our final proposal is shown to exhibit properties akin to the Halfin-Whitt regime.

Tools available

The results have been published in the Performance Evaluation journal, https://www.journals.elsevier.com/performance-evaluation

Next steps and potential cooperation actions in the South American region about the thematic

A number of theoretical questions remain open, regarding the interchange between the diffusion and stationary limits. We also plan to analyze the performance of the feedback control rules when the scaling feature is combined with distributed load balancing mechanisms such as Join-the-Idle-Queue or power-of-d choices.

There are colleagues in the region who have worked on similar tools and applications.

OPTIMAL STOPPING OF STOCHASTIC PROCESSES: THE REPRESENTATION APPROACH

ERNESTO MORDECKI

CENTRO DE MATEMÁTICA DE LA FACULTAD DE CIENCIAS DE LA UNIVERSIDAD DE LA REPÚBLICA, URUGUAY

Content of the presentation

In 1985 Salminen (Mathematische Nachrichten) introduced the representation of excessive functions to solve optimal stopping problems (OSP). Since then, several generalizations have emerged.

In this talk we address: (1) an introduction to general OSP (2) the solution of a standard example, and finally (3) some recent results: an automatic solution for linear diffusions through an algorithm; some multidimensional problems.

Next steps

The method has been explored for general payoffs with regular diffusions, we want to find solutions for general diffusions, with sticky or partial reecting points. Another generalization is the the multimensional case with unbounded continuation region. The last one is the application for problems for diffusions on graphs.

Potential cooperation actions in the South American region about the thematic

Currently there is a PhD student in optimal stopping problems and mean field games. There is a possibility to include master or PhD students in the team of these subjects

COST FUNCTIONALS FOR LARGE RANDOM TREES

JEAN-FRANÇOIS DELMAS

RESEARCH CENTRE IN APPLIED MATHEMATICS (CERMICS), FRANCE

Content of the presentation

In view of the many applications of trees (in computer science, biology, physics, ...), the study of additive functionals on large random trees has seen a lot of development in recent years. We present our results in the so called global regime when the random trees are given by Galton-Watson trees conditioned to be large.

Results available

This work participate to the popularization of the use of random metric spaces to modern mathematics. The results presented are available either as free e-preprint on arXiv or as article in international peer review journals.

- Abraham, Romain; Delmas, Jean-François; Nassif, Michel; Global regime for general additive functionals of conditioned Bienaymé-Galton-Watson trees. <u>Probab. Theory Related Fields</u> 182 (2022), no. 1-2, 277-351. 60380 (05C05 60F17)
- <u>Delmas, Jean-François</u>: <u>Dhersin, Jean-Stéphane</u>; <u>Sciauveau, Marion</u>; Cost functionals for large (uniform and simply generated) random trees. <u>Electron. J. Probab</u>. <u>23 (2018)</u>, Paper No. 87, 36 pp. <u>05C80 (05C05 60F17 60J80)</u>

Next steps

The global regime corresponds to the predominance of the large subtrees whereas the local regime corresponds to the predominance of the small subtrees, and the intermediate regime is in between. The next step would be to consider the intermediate regime for general large Galton-Watson trees.

Potential cooperation actions in the South American region about the thematic

Galton-Watson trees and their continuous analog known as Lévy trees are currently a very dynamic research topic in probability, and is part the recent trend on random

metric spaces (trees, graphs, maps, graphons). We hope this subject could be of interest for further cooperation action in probability.

PORTFOLIO SELECTION UNDER CRAMÉR-LUNDBERG DYNAMICS

JOSÉ CERDA HERNÁNDEZ,

UNIVERSIDAD NACIONAL DE INGENIERÍA, PERU

Content of the presentation

In this work, we examine the combined problem of optimal portfolio selection rules for an insurer in a continuous-time model where the surplus of an insurance company is modelled as a compound Poisson process. The company can invest its surplus in a risk free asset and in a risky asset, governed by the Black-Scholes equation.

According to utility theory, in a financial market where investors are facing uncertainty, an investor is not concerned with wealth maximization per se but with utility maximization. It is therefore possible to introduce an increasing and concave utility function $\Phi(x; t)$ representing the expected utility of a risk averse investor (insurance company).Therefore, the goal of this work is not anymore to maximize the expected portfolio value or minimize the ruin probability or maximizing the expectation of the present value of all dividends paid to the shareholders up to the ruin, but to maximize the expected utility stemming from the wealth during the life contract [0; T], where T is the maturity date of the contract. In this direction, using the Dynamic Programming Principle of the problem, we obtain the Hamilton-Jacobi-Bellman equation by our optimization problem (HJB). Finally, we present numerical solutions in some cases, obtaining as optimal strategy the well known Merton's strategy.

Next steps

Future research would be to study the relationship between the ruin probability using utility theory and the other classical approaches introduced by other authors. This issue needs a further study.

Theoretical results, such as those presented in this paper, help the insurance industry to better manage portfolio risk.

Potential cooperation actions in the South American region about the thematic

Regarding the thematic developed in this work, in the South American region there are small groups that work on risk theory applied to insurance models, for example, Argentina, Chile and Colombia have some researchers working on this thematic. Currently, I have a scholarship with Concytec-Peru. This project included inviting a researcher (working on this topic) from Colombia.



RANDOM POLYMERS IN RANDOM ENVIRONMENT MODEL

ROBERTO VIVEROS,

UNIVERSIDAD DE MINAS GERAIS UFMG, BRAZIL

Content of the presentation

In our work we study the effect of a heavy tailed environment on the phase transition of the Directed Polymer in Random Environment model Our research contributes to the general understanding of disordered systems. This field is one of the most active areas in statistical mechanics and has undergone spectacular development in recent years. Advances on the mathematical side provide a better qualitative understanding of the mechanisms that rule the behavior of physical systems, which can in turn yield new predictions from theoretical physicists.

Tools available

Article: Directed polymer in y-stable random environments

Next steps and potential cooperation actions in the South American region about the thematic

In the literature of directed polymers, very strong disorder (Liapunov exponent of the system being strictly negative) implies strong disorder and a long standing conjecture for directed polymers is that these two critical points actually coincide. However, we exhibited special cases for which there is no very strong disorder phase, while strong disorder holds for sufficiently low temperature. Our project aims to dig further into this phenomenon. To my knowledge there is plenty of cooperation that should be implemented in order to advance the area especially in some countries where the scientific output is still below the regional average.

PROBABILITY, GEOMETRY, TOPOLOGY AND ANALYSIS TO LEARN FROM DATA

PABLO GROISMAN

FACULTAD DE CIENCIAS EXACTAS Y NATURALES DE LA UNIVERSIDAD DE BUENOS AIRES, ARGENTINA

Content of the presentation

A new proposal to infer density-based distances in a manifold from a sample, called Fermat distance, has been discussed. With this distance at our disposal we can outperform standard methods for clustering, classification and topological data analysis.

Tools available

- An implementation of Fermat is available at <u>https://www.aristas.com.ar/fermat/index.html</u> for its use in clustering procedures and other tasks.
- In https://github.com/ximenafernandez/intrinsicPH you can find the code for the use of Fermat for topology learning and to find patterns in chaotic dynamical systems.

Next steps and potential cooperation actions in the South American region about the thematic

We plan to continue working on applications of this distance to infer properties of data sets. We are also working to obtain rigorous results for the behavior of this metric in the presence of noise. We have collaboration with groups in Uruguay, Chile and Brasil.



STOCHASTIC CONTROL AND DIFFERENTIAL GAMES WITH PATH-DEPENDENT INFLUENCE OF CONTROL TRAJECTORY ON DYNAMICS AND RUNNING COST

YURI SAPORITO ESCUELA DE MATEMÁTICA APLICADA DE LA FUNDACIÓN GETULIO VARGAS, BRASIL

Content of the presentation

This presentation showed a novel application of Functional Itô Calculus to the problem of optimal stochastic control under the situation that the path of the control chosen by the agent impacts her dynamics and running cost/gain. The delayed case served as an example.

Tools available

Related paper https://locus.siam.org/doi/abs/10.1137/18M1186186? mobileUi=0&

Next steps and potential cooperation actions in the South American region about the thematic

Future work could involve new applications for this type of pathdependence or the theoretical study of viscosity solution for the new HJB that arose from the theory.

STOCHASTIC EPIDEMIC MODELS WITH RANDOM PERTURBATIONS

VISWANATHAN ARUNACHALAM DEPARTMENT OF STATISTICS, UNIVERSIDAD NACIONAL DE COLOMBIA

Content of the presentation

This talk presents a study of stochastic epidemic models with random perturbations on disease transmission rates influenced by demographic and environmental stochasticity factors. We discuss the method of Maximum likelihood Estimation (MLE) and Mean-Squared Error (MSE) for the random perturbation parameter (σ), also called a volatility parameter. Finally, we apply our model to study and illustrate the measures of the infectiousness of the COVID-19 dynamic in Bogotá.

Tools available

1.Ríos-Gutiérrez, A., Torres, S. and Arunachalam, V., 2021. Studies on the basic reproduction number in stochastic epidemic models with random perturbations. Advances in difference equations, 2021(1), pp.1-24.

2.Niño-Torres, D., Ríos-Gutiérrez, A., Arunachalam, V., Ohajunwa, C. and Seshaiyer, P., 2021. Stochastic modeling, analysis, and simulation of the COVID-19 pandemic with explicit behavioral changes in Bogotá: A case study. Infectious Disease Modelling.

Next steps and potential cooperation actions in the South American region about the thematic

The development of new vaccines to control the spread of COVID-19 and the implementation and effectiveness of vaccination programs poses several unique challenges. Our future research interest is to study the effects of implementing vaccines with different efficacies for preventing the spread of the diseases in the South-American regions with different socio-economic characteristics.

we recently published a research article and currently studying the recent COVID-19 pandemic in collaboration with colleagues from Chile. As part of The MATH AmSud project, we plan to organize a workshop on the thematic to strengthen future cooperation between the scientific community.



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The seminar was organized by:







The MATH AmSud Program is a joint cooperation action between:

